

# A Tractable Method for Chance-Constrained Power Control in Downlink Multiuser MISO Systems with Channel Uncertainty

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**Abstract**—We consider a downlink wireless system with a multi-antenna base station (BS) and single-antenna users. The error in the channel knowledge at the BS is assumed to have Gaussian distribution. Power allocation strategies are designed in order to satisfy the users' quality-of-service targets with certain probabilities. Conservative solutions of the problems are found by applying the Vysochanskii-Petunin inequality in combination with the theory of interference functions. Significant performance improvements are obtained comparing with methods based on the worst-case optimization.

**Index Terms**—Multiple-antenna systems, imperfect channel knowledge, stochastic programming, interference functions.

## I. INTRODUCTION

It is known that downlink precoding in wireless communication systems can be quite sensitive with respect to the channel state information (CSI) at the transmitter [1]. Providing quality-of-service (QoS) constrained transceiver designs which are robust to the imperfect CSI has attracted a lot of attention recently. Basically, two approaches have emerged. In the worst-case strategies, it is assumed that the CSI errors are bounded. Certain performance targets are guaranteed then for all channels from the uncertainty regions (see, e.g., [2]). On the other hand, stochastic methods suppose that the CSI errors exhibit certain statistical properties, which is often the case in estimation and quantization procedures. Generally, the statistical information about the CSI mismatch can be used to optimize the mean or the outage performance of the system. Notice that for unbounded CSI errors, a desired QoS performance can often be promised only with a certain probability. These issues, combined with the fact that the worst-case optimization might be overly conservative in practice, lead to the concept of chance (or probabilistically) constrained signal processing [3].

The main contribution of this paper is an algorithm for robust power allocation under probabilistic QoS targets, in a downlink multiple-input single-output (MISO) system with the Gaussian CSI mismatch. Provided with the error distribution, it is known that the size of the uncertainty region for the worst-case methods can be calculated for this system, so

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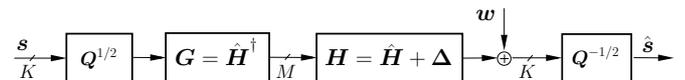


Fig. 1. Downlink, power-controlled, multiuser MISO system.

that the QoS targets are satisfied with the desired probability [4]. We show that this worst-case based approach can be significantly outperformed for a wide range of scenarios, by exploiting the problem structure deeper. The power control optimization is divided in two parts. First, we eliminate the stochastic uncertainty in a conservative manner, by applying the Vysochanskii-Petunin inequality [5]. The obtained deterministic optimization problem is then solved using the theory of interference functions [6]. We remark that the idea for applying the Vysochanskii-Petunin inequality to obtain conservative approximations in stochastic programming appeared recently also in [7]. However, the motivation behind the work [7], the mathematical structure of the problem, and the applied optimization technique, are considerably different from the problems studied in this paper.

## II. SYSTEM MODEL AND PROBLEM STATEMENT

The considered multiuser, flat-fading MISO system is illustrated in Fig. 1. The base station (BS) is equipped with  $M$  antennas. It transmits vector  $\mathbf{s} = [s_1, \dots, s_K]^T \in \mathbb{C}^K$ , where the symbol  $s_k$  is intended for the  $k$ th single-antenna user,  $k = 1, \dots, K$ . There exists no cooperation among users. We denote the complete downlink channel with  $\mathbf{H} \in \mathbb{C}^{K \times M}$ , where the  $k$ th row of  $\mathbf{H}$ ,  $\mathbf{H}_{(k,:)}$  (this notation for a row of a matrix is used throughout the paper) is the  $k$ th user's channel. The BS is provided only with an estimate  $\hat{\mathbf{H}}$  of  $\mathbf{H}$ , where  $\hat{\mathbf{H}}$  has the full row rank. The CSI error matrix is defined as  $\Delta = \mathbf{H} - \hat{\mathbf{H}}$ . It is assumed that  $\Delta_{(k,:)} = \bar{\Delta}_{(k,:)} \mathbf{C}_k$ , where  $\bar{\Delta}_{(k,m)}$ ,  $m = 1, \dots, M$  are i.i.d. complex Gaussian random variables (RVs) with independent real and imaginary parts having  $\mathcal{N}(0, 1/2)$ -distribution, and  $\mathbf{C}_k \in \mathbb{C}^{M \times M}$ . As an application example, notice that the Gaussian error model corresponds well to time-division-duplex (TDD) systems, where the estimated channel coefficients at the BS from the uplink phase are used for the downlink precoding.

The linear precoder of the BS is composed of two parts. We optimize the diagonal power control matrix  $\mathbf{Q}^{1/2} = \text{diag}(\sqrt{q_1}, \dots, \sqrt{q_K})$ . The beamforming matrix  $\mathbf{G} \in \mathbb{C}^{M \times K}$  is fixed, with  $\mathbf{G} = \hat{\mathbf{H}}^\dagger$ , i.e., the Moore-Penrose pseudoin-

version (zero-forcing) with respect to the available imperfect CSI is performed. The users equalize the received signals by a multiplication with  $q_k^{-1/2}$ , so the system equations are

$$s_k = q_k^{-1/2} \mathbf{H}_{(k,:)} \mathbf{G} \mathbf{Q}^{1/2} \mathbf{s} + w_k, \quad k = 1, \dots, K. \quad (1)$$

We remark that a similar system model is analyzed with the goal of worst-case optimization in [4]. As QoS measures, we adopt the MSEs between the sent and the signals after the equalization

$$\text{MSE}_k = \text{E}\{|s_k - \hat{s}_k|^2\}, \quad k = 1, \dots, K. \quad (2)$$

For the transmit vector  $\mathbf{s}$  and the receive noise  $\mathbf{w} = [w_1, \dots, w_K]^T$ , it is supposed that  $\text{E}\{\mathbf{s}\mathbf{s}^*\} = \mathbf{I}$ ,  $\text{E}\{\mathbf{w}\mathbf{w}^*\} = \text{diag}(\sigma_1^2, \dots, \sigma_K^2)$ , where  $(\cdot)^*$  denotes the complex conjugation. The total transmit power  $P$  is the sum of the scaled elements of  $\mathbf{Q}$ , and it can be calculated as  $P = \text{Tr}(\mathbf{G}\mathbf{Q}\mathbf{G}^*)$ . We define the principal problem of interest as the minimization of the total transmit power, subject to fixed MSE targets  $\mu_k \in (0, 1)$ , that should be satisfied with probabilities  $p_k$

$$\min_{\mathbf{Q}} P \quad \text{s.t.} \quad P \leq P_{\max}, \quad \text{Pr}\{\text{MSE}_k \leq \mu_k\} \geq p_k, \quad \forall k. \quad (3)$$

We assume a block fading scenario, and the probability in (3) refers to the channel uncertainty. As usual in practical systems, the solution of (3) should also obey a total transmit power constraint  $P \leq P_{\max}$ .

### III. ELIMINATION OF STOCHASTIC UNCERTAINTY

In this section, the focus is on transforming the chance constraints in (3) into tractable deterministic problems. The fact that our QoS measures of interest are quadratic functions makes the probabilistic optimization problem (3) intricate. For the zero-forcing beamformer  $\mathbf{G}$  specified in the previous section, the MSE of the  $k$ th user can be calculated as  $\text{MSE}_k = q_k^{-1} \bar{\Delta}_{(k,:)} \mathbf{G} \mathbf{Q} \mathbf{G}^* \bar{\Delta}_{(k,:)}^* + q_k^{-1} \sigma_k^2$ . Therefore, the condition  $\text{MSE}_k \leq \mu_k$  can be rewritten as

$$X_k \triangleq \bar{\Delta}_{(k,:)} \mathbf{G} \mathbf{Q} \mathbf{G}^* \bar{\Delta}_{(k,:)}^* \leq \mu_k q_k - \sigma_k^2 \quad (4)$$

where  $\mathbf{G}_k \triangleq \mathbf{C}_k \mathbf{G}$ .  $X_k$  is a quadratic form in normal variables, and the expression for its probability density function (PDF) is known to be involved [8]. Therefore, we will derive a conservative tractable approximation of (3), in the sense of obtaining deterministic constraints that imply the stochastic constraints in (3). Approximations for more general quadratic constraints than (4) were proposed also in [9]. We will adopt another methodology to exploit the special structure in (4) and obtain simpler deterministic problems with an improved performance. It should be remarked that the optimal solution for a related, but mathematically different, worst-case PDF problem from [9], could be applied in our scenario with minor modifications.

First, we prove that  $X_k$  is unimodal. In other words, there exists at least one number  $m \in \mathbb{R}$ , so that the PDF of  $X_k$  is non-decreasing on  $[0, m)$  and non-increasing on  $(m, \infty)$  [5]. Perform a unitary diagonalization  $\mathbf{G}_k \mathbf{Q} \mathbf{G}_k^* = \mathbf{U}_k \mathbf{\Lambda}_k \mathbf{U}_k^*$ , with unitary  $\mathbf{U}_k \in \mathbb{C}^{M \times M}$  and  $\mathbf{\Lambda}_k = \text{diag}(\lambda_{k,1}, \dots, \lambda_{k,M})$ ,  $\lambda_{k,1}, \dots, \lambda_{k,M} \geq 0$ . It can be seen that

$d_k = [D_{k,1}, \dots, D_{k,M}]^T \triangleq \mathbf{U}_k^* \bar{\Delta}_{(k,:)}^* \in \mathbb{C}^M$  is a vector of i.i.d. zero-mean complex Gaussian variables with independent real and imaginary parts. The standard quadratic form for  $X_k$  is then  $X_k = \sum_{m=1}^M \lambda_{k,m} (\Re\{D_{k,m}\}^2 + \Im\{D_{k,m}\}^2)$ , where  $\Re\{\cdot\}$  and  $\Im\{\cdot\}$  extract the real and imaginary parts of the argument, respectively. The unimodality of this sum for any  $\lambda_{k,1}, \dots, \lambda_{k,M} \geq 0$  follows directly from Theorem 4 in [10].

$X_k$  can be expressed using the real-valued system representation as a sum of correlated gamma variables

$$X_k = \sum_{l=1}^{2K} q_{\kappa(l)} \left( \tilde{\mathbf{G}}_{k(l,:)} \tilde{\boldsymbol{\delta}}_k \right)^2 \quad (5)$$

where  $\kappa(l) = l$  for  $l = 1, \dots, K$ , and  $\kappa(l) = l - K$  for  $l = K + 1, \dots, 2K$ ,

$$\tilde{\mathbf{G}}_k = \begin{bmatrix} \Re\{\mathbf{G}_k^*\} & -\Im\{\mathbf{G}_k^*\} \\ \Im\{\mathbf{G}_k^*\} & \Re\{\mathbf{G}_k^*\} \end{bmatrix}, \quad \tilde{\boldsymbol{\delta}}_k = \begin{bmatrix} \Re\{\bar{\Delta}_{(k,:)}^*\} \\ \Im\{\bar{\Delta}_{(k,:)}^*\} \end{bmatrix}. \quad (6)$$

The mean and the variance of  $X_k$  can be calculated as

$$\text{E}\{X_k\} = \sum_{l=1}^{2K} q_l \|\mathbf{G}_{k(\cdot,l)}\|_2^2 \quad (7)$$

$$\text{Var}\{X_k\} = \frac{1}{2} \sum_{i=1}^{2K} \sum_{j=1}^{2K} q_{\kappa(i)} q_{\kappa(j)} \left( \tilde{\mathbf{G}}_{k(i,:)} \tilde{\mathbf{G}}_{k(j,:)}^T \right)^2 \quad (8)$$

where  $\mathbf{G}_{k(\cdot,l)}$  denotes the  $l$ th column of  $\mathbf{G}_k$ .

For the further analysis, we derive a one-sided form of the Vysochanskii-Petunin inequality. Let  $X$  be a unimodal RV,  $a \in \mathbb{R}_{++}$  and  $y \leq \text{E}\{X\}$  a deterministic parameter. It holds that

$$\begin{aligned} \text{Pr}\{X - \text{E}\{X\} \geq a\} &\leq \text{Pr}\{|X - y| \geq a + \text{E}\{X\} - y\} \\ &\leq \max \left\{ \frac{4}{9} \frac{\text{E}\{(X - y)^2\}}{(a + \text{E}\{X\} - y)^2}, \frac{4}{3} \frac{\text{E}\{(X - y)^2\}}{(a + \text{E}\{X\} - y)^2} - \frac{1}{3} \right\} \end{aligned} \quad (9)$$

where the last relation is the two-sided Vysochanskii-Petunin inequality [5]. Since  $\text{E}\{(X - y)^2\} = \text{Var}\{X\} + (\text{E}\{X\} - y)^2$ , and the last term in (9) is minimized for  $y = \text{E}\{X\} - \text{Var}\{X\}/a$ , we conclude that

$$\begin{aligned} \text{Pr}\{X - \text{E}\{X\} \geq a\} \\ \leq \max \left\{ \frac{4}{9} \frac{\text{Var}\{X\}}{\text{Var}\{X\} + a^2}, \frac{4}{3} \frac{\text{Var}\{X\}}{\text{Var}\{X\} + a^2} - \frac{1}{3} \right\}. \end{aligned} \quad (10)$$

We can now formulate the conservative, deterministic approximations of the constraints in (3). Let  $f(p_k) = \sqrt{(1 - p_k)/(p_k - 5/9)}$  for  $p_k \geq 5/6$ , and  $f(p_k) = \sqrt{(4/3 - p_k)/p_k}$  for  $p_k < 5/6$ . We claim that the probabilistic constraints in (3) are implied by

$$f(p_k) (\mu_k q_k - \sigma_k^2 - \text{E}\{X_k\}) \geq \sqrt{\text{Var}\{X_k\}}, \quad k = 1, \dots, K. \quad (11)$$

Let  $a_k = \mu_k q_k - \sigma_k^2 - \text{E}\{X_k\}$ . Notice that (11) implies  $a_k > 0$ . Using (10), we obtain

$$\begin{aligned} \text{Pr}\{X_k \leq \mu_k q_k - \sigma_k^2\} &= 1 - \text{Pr}\{X_k - \text{E}\{X_k\} \geq a_k\} \\ &\geq 1 - \max \left\{ \frac{4}{9} \frac{\text{Var}\{X_k\}}{\text{Var}\{X_k\} + a_k^2}, \frac{4}{3} \frac{\text{Var}\{X_k\}}{\text{Var}\{X_k\} + a_k^2} - \frac{1}{3} \right\}. \end{aligned} \quad (12)$$

If the last term in (12) is greater or equal to  $p_k$ , the safe approximation is obtained in the form of two constraints that must be fulfilled

$$\frac{\text{Var}\{X_k\}}{\text{Var}\{X_k\} + a_k^2} \leq \frac{9}{4}(1 - p_k), \quad \frac{\text{Var}\{X_k\}}{\text{Var}\{X_k\} + a_k^2} \leq 1 - \frac{3}{4}p_k. \quad (13)$$

For  $p_k \geq 5/6$ , the right hand-side term of the first condition in (13) is smaller than the right hand-side term of the second condition in (13). In this case,

$$a_k \sqrt{(1 - p_k)/(p_k - 5/9)} \geq \sqrt{\text{Var}\{X_k\}} \quad (14)$$

clearly implies both conditions in (13), and, correspondingly, the probabilistic MSE constraint for the user  $k$  in (3). For  $p_k < 5/6$ , the second equation in (13) is more critical. The probabilistic MSE constraint in (3) is conservatively approximated in this case by  $a_k \sqrt{(4/3 - p_k)/p_k} \geq \sqrt{\text{Var}\{X_k\}}$ .

#### IV. ITERATIVE OPTIMIZATION

Notice that the objective function in (3) can be rewritten as  $\sum_{k=1}^K q_k \|\mathbf{G}_{(:,k)}\|_2^2$ . Introduce new variables  $\mathbf{t} = [t_1, \dots, t_K]^T$ ,  $t_k = q_k \|\mathbf{G}_{(:,k)}\|_2^2$ . The conservative approximation of the problem (3) is then

$$\min_{\mathbf{t}} \sum_{l=1}^K t_l \quad \text{s.t.} \quad \sum_{l=1}^K t_l \leq P_{\max}, \quad \frac{t_k}{\mathcal{I}_k(\mathbf{t})} \geq \beta_k, \quad \forall k \quad (15)$$

where  $\beta_k = \|\mathbf{G}_{(:,k)}\|_2^2 \sigma_k^2 / \mu_k$ ,

$$\mathcal{I}_k(\mathbf{t}) = 1 + \frac{1}{\sigma_k^2} \sum_{l=1}^K t_l \psi_{k,l} + \frac{1}{\sigma_k^2 f(p_k)} \sqrt{\sum_{l=1}^K t_l^2 \psi_{k,l}^2 + \sum_{i=1}^{2K-1} \sum_{j=i+1}^{2K} t_{\kappa(i)} t_{\kappa(j)} \eta_{k,i,j}}, \quad (16)$$

$$\psi_{k,l} = \frac{\|\mathbf{G}_{k(i,l)}\|_2^2}{\|\mathbf{G}_{(:,l)}\|_2^2}, \quad \eta_{k,i,j} = \frac{\left(\tilde{\mathbf{G}}_{k(i,:)} \tilde{\mathbf{G}}_{k(j,:)}^T\right)^2}{\|\mathbf{G}_{(:,\kappa(i))}\|_2^2 \|\mathbf{G}_{(:,\kappa(j))}\|_2^2}. \quad (17)$$

Let  $\mathcal{I}(\mathbf{t}) = [\mathcal{I}_1(\mathbf{t}), \dots, \mathcal{I}_K(\mathbf{t})]^T$ . It can be easily checked that  $\mathcal{I}(\mathbf{t})$  satisfies the following properties

$$\mathcal{I}(\mathbf{t}) > \mathbf{0} \quad (18)$$

$$\mathbf{t} \geq \boldsymbol{\tau} \Rightarrow \mathcal{I}(\mathbf{t}) \geq \mathcal{I}(\boldsymbol{\tau}) \quad (19)$$

$$\alpha \mathcal{I}(\mathbf{t}) > \mathcal{I}(\alpha \mathbf{t}), \quad \forall \alpha > 1 \quad (20)$$

where the inequalities between vectors are component-wise. Therefore,  $\mathcal{I}_k(\mathbf{t})$ ,  $k = 1, \dots, K$ , can be classified as *standard interference functions*, and the fixed point iteration

$$\mathbf{t}^{(n+1)} = \text{diag}(\beta_1, \dots, \beta_K) \mathcal{I}(\mathbf{t}^{(n)}) \quad (21)$$

converges to the optimal solution of the problem (15) for any initial vector  $\mathbf{t}$ , if (15) is feasible [6].

To handle the feasibility issue, we start with  $\mathbf{t}^{(0)} = \mathbf{0}$ . Notice that (18) implies that  $\mathcal{I}(\mathbf{0}) > \mathbf{0}$ . Furthermore, it holds that  $\mathbf{t}^{(1)} = \text{diag}(\beta_1, \dots, \beta_K) \mathcal{I}(\mathbf{0}) > \mathbf{t}^{(0)} = \mathbf{0}$ . Using (19), we obtain  $\mathbf{t}^{(2)} = \text{diag}(\beta_1, \dots, \beta_K) \mathcal{I}(\mathbf{t}^{(1)}) \geq \text{diag}(\beta_1, \dots, \beta_K) \mathcal{I}(\mathbf{t}^{(0)}) = \mathbf{t}^{(1)}$ . The procedure can be continued in the same manner, so the initialization  $\mathbf{t}^{(0)} = \mathbf{0}$  in (21) yields  $\mathbf{t}^{(n+1)} \geq \mathbf{t}^{(n)}$ ,  $n \geq 1$ .

Suppose now that the problem (15) is infeasible, and that there is a convergence point  $\mathbf{t}^*$  of the fixed-point iteration (21) with  $\sum_{k=1}^K t_k^* \leq P_{\max}$ . If this is true, due to the continuity of the interference functions  $\mathcal{I}_k(\mathbf{t})$  in (16), we have

$$\begin{aligned} t_k^* &= \lim_{n \rightarrow \infty} t_k^{(n)} = \lim_{n \rightarrow \infty} t_k^{(n+1)} \\ &= \lim_{n \rightarrow \infty} \beta_k \mathcal{I}_k(\mathbf{t}^{(n)}) = \beta_k \mathcal{I}_k(\mathbf{t}^*) \end{aligned} \quad (22)$$

for  $k = 1, \dots, K$ . In other words,  $\mathbf{t}^*$  is a feasible solution, which is a contradiction. Therefore, we conclude that there can exist no finite convergence point if the problem domain is empty due to the constraints  $t_k / \mathcal{I}_k(\mathbf{t}) \geq \beta_k$ .

To summarize, for  $\mathbf{t}^{(0)} = \mathbf{0}$ , the iterative procedure (21) yields a component-wise monotonically increasing sequence  $\mathbf{t}^{(n)}$ . If this sequence converges to a point before exceeding the total transmit power constraint, the convergence point is the optimal solution. Otherwise, the infeasibility can be immediately declared.

#### A. Related Problems and Extensions

The method presented above can be easily modified to cover several interesting related problems, such as the QoS targets in terms of signal-to-interference-plus-noise-ratios (SINRs), or the min-max fairness problem.

Minimum tolerable SINR targets present often a more convenient performance measure than the MSEs. From Theorem 3 in [11], it can be concluded that guaranteeing certain MSE targets  $\mu_k$  under uncertainty assures that the SINR targets  $\gamma_k = \mu_k^{-1} - 1$  are fulfilled with the same transmit filter, as well (notice that for SINRs, the users' equalization plays no role). In this way, an SINR-constrained problem of minimizing the total transmit power subject to probabilistic constraints  $\Pr\{\text{SINR}_k \geq \gamma_k\} \geq p_k$  can be conservatively approached by defining a virtual MSE-constrained problem (3) with  $\mu_k = (\gamma_k + 1)^{-1}$ .

In the min-max fairness problem, one is interested in minimizing the "balanced" level of the users' MSEs  $\mu$ , having in mind that a certain outage probability is allowed

$$\min_{\mathbf{Q}, \mu} \mu \quad \text{s.t.} \quad \Pr\{\text{MSE}_k \leq \mu\} \geq p_k, \quad \forall k, \quad P \leq P_{\max}. \quad (23)$$

This problem might be of particular practical importance, since the goal of the system design is often to minimize the error under some given power budget. The problem (23) can be solved using the bisection with respect to the level  $\mu$ . The starting search interval for  $\mu$  is  $[0, 1]$ . In each step of the bisection procedure, for a given  $\mu$ , it should be determined whether (23) is feasible. This can be done using the feasibility analysis described above. Notice that, contrary to (3), the complete problem (23) can never be infeasible.

Finally, we remark that the interference functions (16) have a very rich structure, including convexity and log-convexity (see [12] for the definition of log-convex interference functions), which can be exploited for improving the convergence properties of (21) and defining other iterative algorithms for solving (15). These issues will be addressed in a future, more comprehensive work on this topic.

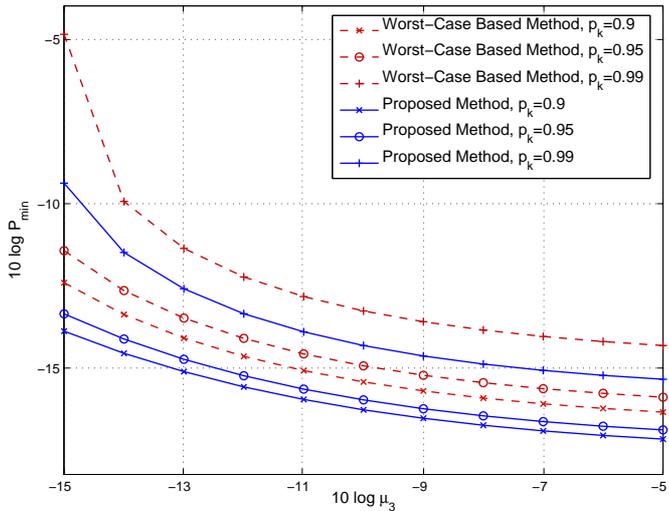


Fig. 2. Minimal transmit power vs. the MSE target  $\mu_3$ , with  $\mu_1 = \mu_2 = 0.1$ .

## V. NUMERICAL EXAMPLES

We consider firstly a system with parameters  $M = K = 3$ ,  $\sigma_k^2 = 10^{-3}$ ,  $\mathbf{C}_k = 2\sigma_k \mathbf{I}$ ,  $\forall k$ ,  $\mu_1 = \mu_2 = 0.1$ . In Fig. 2, the average minimum transmit power against the target  $\mu_3$  is plotted, for three scenarios regarding the probabilistic constraints:  $p_k = 0.99$ ,  $\forall k$ ,  $p_k = 0.95$ ,  $\forall k$ , and  $p_k = 0.9$ ,  $\forall k$ . The results present an average over  $10^3$  feasible erroneous channels. The erroneous channel coefficients were generated as complex normal variables with zero mean and unit variance. The performance is compared with the worst-case based approach. In this method, under the assumptions stated above, the radius of the ball uncertainty region, that guarantees the targets with the probability  $p_k$ , can be simply calculated as  $\varepsilon_k = \sqrt{2}\sigma_k \sqrt{F_{\chi^2, 2M}^{-1}(p_k)}$ , with  $F_{\chi^2, 2M}$  being the cumulative density function of the central chi-square distribution with  $2M$  degrees of freedom. The optimal solution for the worst-case power control from [4] can be applied then. It can be noticed that the method proposed in this paper significantly outperforms the worst-case based strategy. The number of iterations for the termination of the fixed-point iteration algorithm monotonically decreased with the growing MSE target  $\mu_3$ , from 6.48 to 4.99, 7.56 to 5.53 and 17.53 to 8.23, for the specified probabilities  $p_k \in \{0.90, 0.95, 0.99\}$ , respectively. The relative difference of  $10^{-3}$  for the norm of  $\mathbf{t}$  was taken as the stopping criterion.

In the second experiment, we fixed  $p_k = 0.95$ ,  $\mu_k = 0.1$ ,  $k = 1, \dots, K$ , and counted the number of feasible channel realizations for the problem (15) in systems with  $M = K \in \{2, \dots, 6\}$ . Other system parameters were the same as in the previous example. The percentages of feasible channels for the method proposed in this paper were 75.43%, 68.05%, 60.89%, 54.17%, 48.01%, for  $M = K = 2, \dots, 6$ , respectively. The worst-case approach yielded 68.47%, 46.64%, 28.44%, 16.47%, 8.25% feasible channels, respectively, for the same parameters.

We conclude this section by showing gains in comparison with the worst-case strategy for the min-max fairness problem

$\sigma_k^2/10^{-3}$	1	2	4	8
Worst-Case Based Method	0.0193	0.0325	0.0651	0.1301
Proposed Method	0.0106	0.0209	0.0417	0.0832

TABLE I  
MIN-MAX MSE  $\mu$  IN A 3-USER MISO SYSTEM WITH  $p_k = 0.95$ .

(23). The average value of the min-max level  $\mu$  is given in Table I, for a system with  $M = K = 3$ , and  $p_k = 0.95$ ,  $\sigma_k^2 = 10^{-3}$ ,  $\mathbf{C}_k = \sqrt{2}\sigma_k \mathbf{I}$ ,  $\forall k$ .

We report also that only in “small” systems of size  $M \leq 3$ ,  $1 < K \leq M$ , with very high probabilities  $p_k \rightarrow 1$  (these are often impractical because the feasibility region becomes considerably smaller), the worst-case approach exhibited a somewhat better performance. In all other scenarios we have investigated, both regarding the system size and the probabilities  $p_k$ , our method has yielded significant gains.

## VI. CONCLUSION

A power control method, robust to imperfect transmit CSI, is proposed for a downlink MISO system with probabilistic QoS targets. The conservative approximation of the main problem, obtained by applying the Vysochanskii-Petunin inequality, is iteratively solved using the interference functions framework. Besides the MSE-constrained power minimization problem, SINR targets and the min-max fairness problem are supported, as well. The solutions outperform the worst-case methods for the scenarios of practical interest and have low complexity.

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